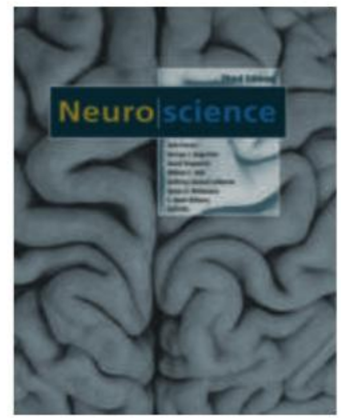
# Passive Membrane Properties

Communicating through a leaky garden hose....

| Topics I | Topics II |
|---|---|
| Introduction & Electrochemical Gradients | Synaptic Transmission |
| Passive Membrane Properties | Electrophysiology Techniques |
| Action Potentials | Basic Circuits (Spinal Cord) |
| Voltage-Gated Ion Channels | Sensory Systems Overview |
| Ligand-Gated Ion Channels | Synaptic Plasticity |

# Study Material

- NEUROSCIENCE Third Edition
  - – Dale Purves
- Chapter 3 pages 58-61
- (The textbook is not very detailed about these things)




**THE COVER**
Dorsal view of the human brain.
(Courtesy of S. Mark Williams.)

NEUROSCIENCE: Third Edition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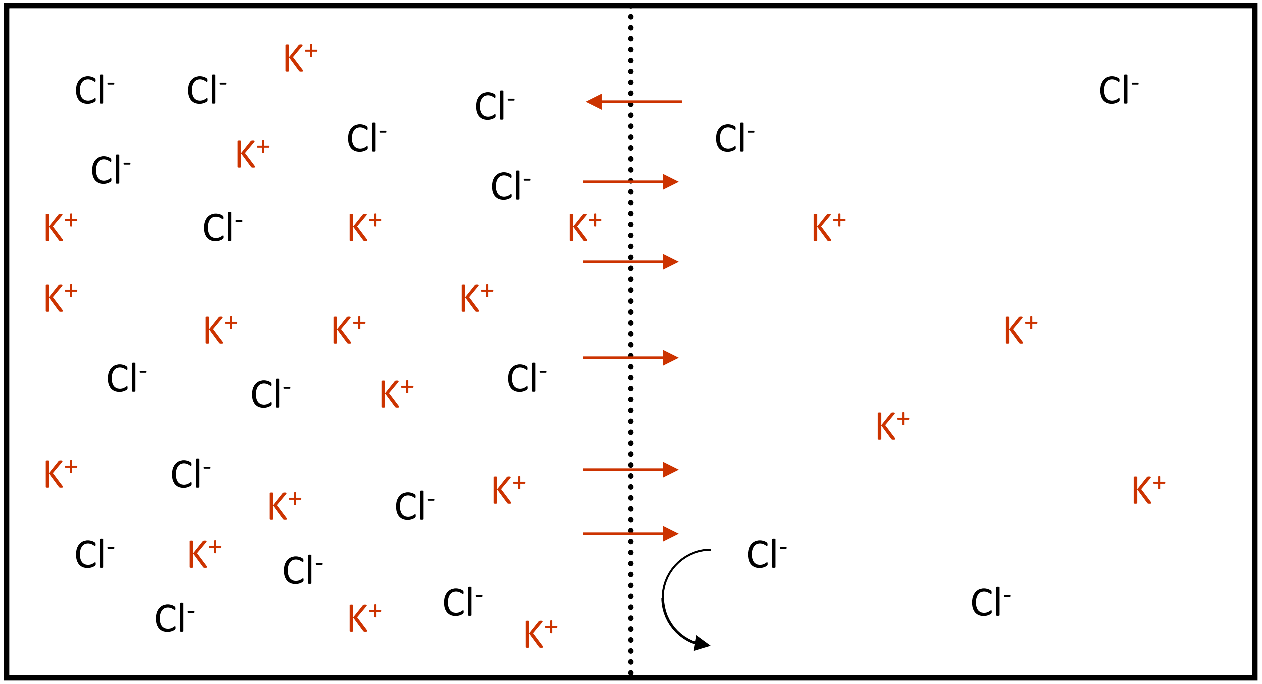
# Aims

- Understand the cell membrane as a capacitor and a resistor.
- Understand that the capacitor acts as an ‚elastic‘ element that stores charge.
- Know how to calculate the membrane time constant.
- Understand that the time constant limits the speed with which the membrane voltage can be changed.
- Understand how membrane resistance and capacitance affect the propagation of signals in neuronal processes.

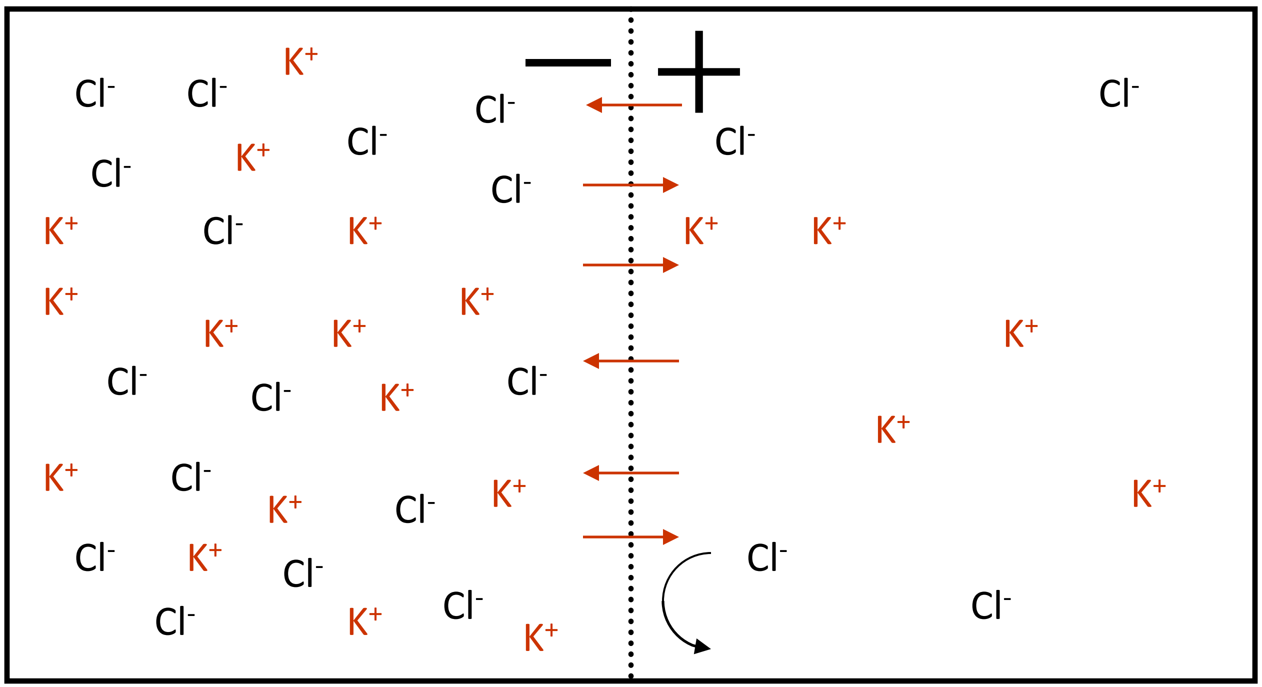
# Recapitulation L1

- Ion concentrations are very different inside and outside of neurons. The inside is rich in K and poor in Na. The outside is rich in Na and poor in K.
- Coupled with selective permeability these gradients produce electrical potentials.
- At rest the dominant permeability is for K ions, the resulting potential is around -60 mV (inside relative to outside the neuron).

# Recapitulation



# Recapitulation



# Recapitulation

Electrical Field

$$nzF(E_i - E_o)$$

Chemical Energy

$$nRT \ln \frac{[K^+]_o}{[K^+]_i}$$

| | |
|---|---|
| n | Amount (Mol) |
| z | Valence (+1 for K) |
| F | Faraday‘s constant (charge of one Mole of Ions) |
| E | Potential |
| R | Gas constant |
| T | Temperature (absolute K) |
| [ ] | Concentration in the respective compartment |

# Recapitulation

$$zF(E_i - E_o) = RT \ln \frac{[K^+]_o}{[K^+]_i}$$

$$E_i = \frac{RT}{zF} \ln \frac{[K^+]_o}{[K^+]_i}$$

Nernst Equation
Describes equilibrium
No **net** flux

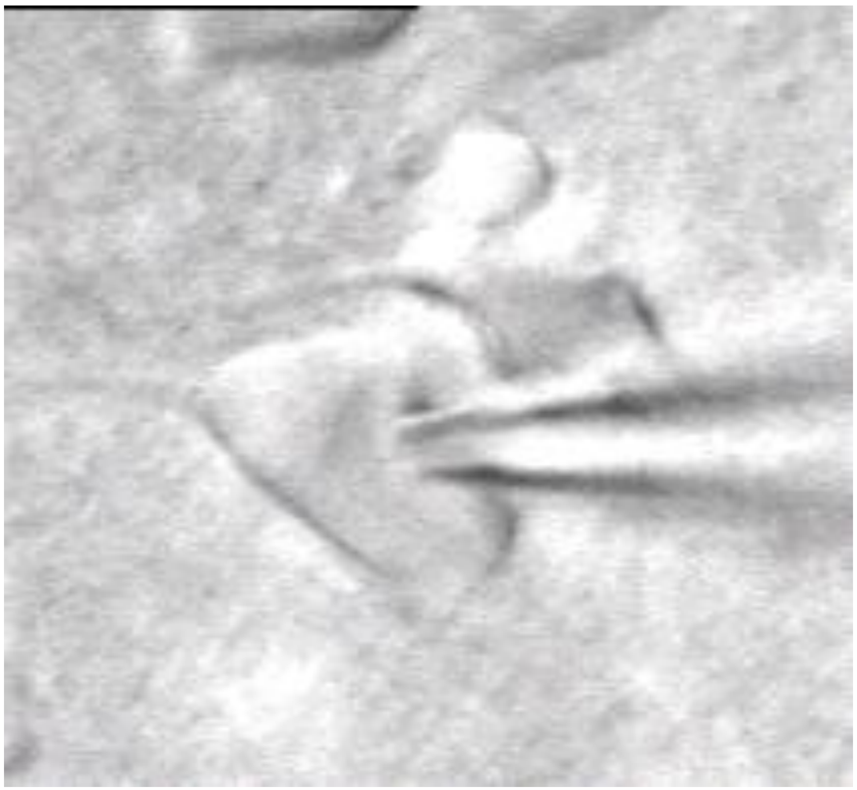
# Recapitulation

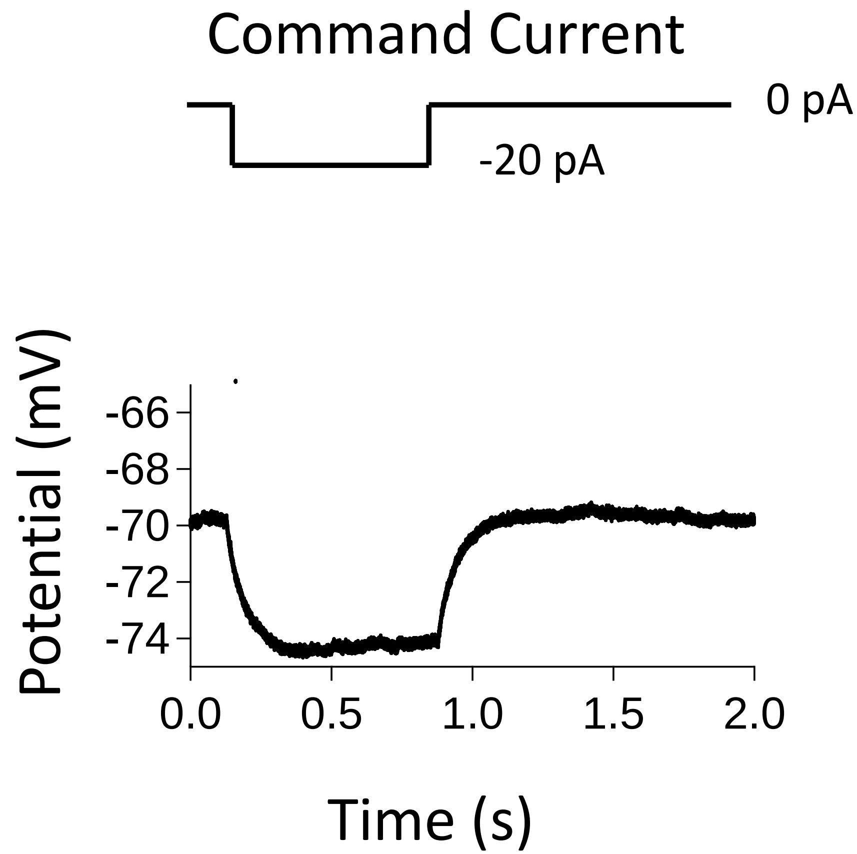
$$E_i = \frac{RT}{zF} \ln \frac{[K^+]_o}{[K^+]_i}$$

$$E_i = 61(mV) \log \frac{[K^+]_o}{[K^+]_i}$$

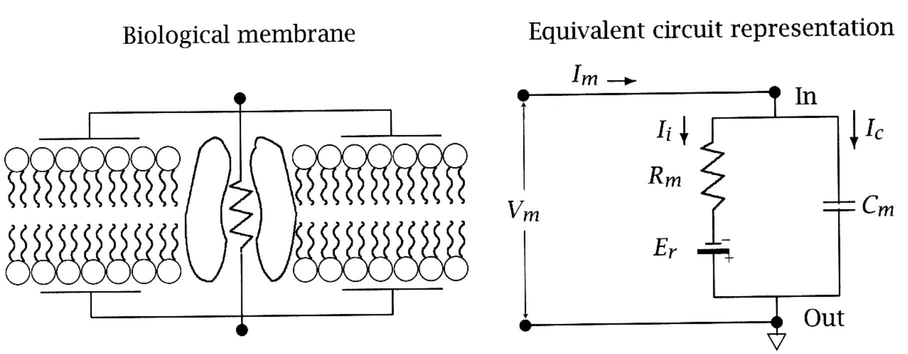
| | Intracellular | Extracellular | E |
|---|---|---|---|
| Potassium Ions | 155 mM | 4 mM | -98 mV |
| Sodium Ions | 12 mM | 145 mM | +67mV |

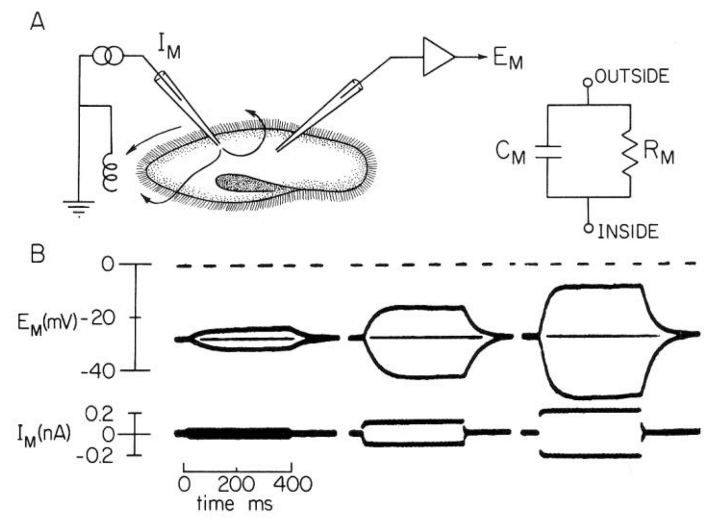
# Passive Membrane Properties





# The Electrical Circuit




Cell membranes are acting as capacitors and resistors.

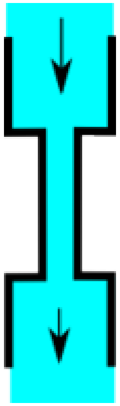
# Hydraulic Analogy

**Electrical Property**

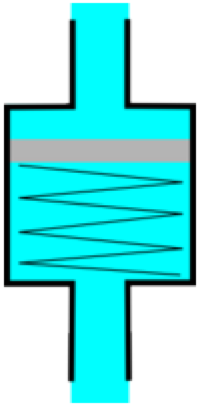
- Voltage
- Charge
- Current
- Resistor
- Capacitor

**Hydraulic Property**

- Pressure
- Water Volume
- Flow of Water
- Control Valve
- Elastic Container



Resistor

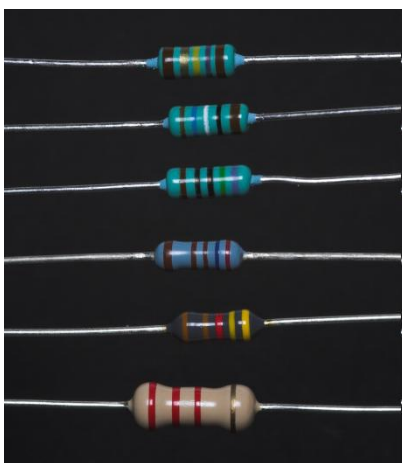


Capacitor

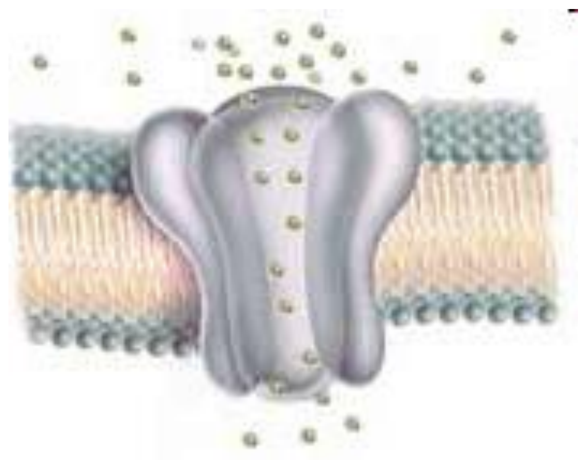
# Ohmic Resistor

Linear relationship between current through resistor (I) and voltage across resistor (U)

$$R = \frac{U}{I}$$



Electronics



Biology

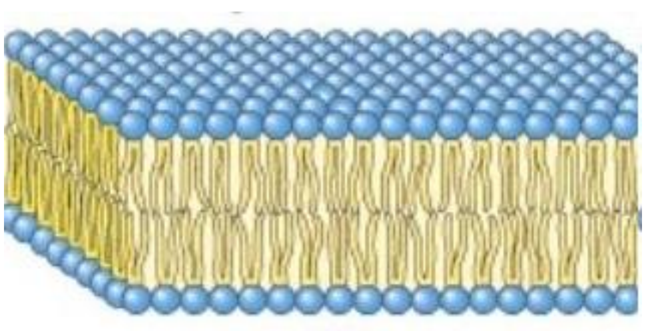
$$U = R \cdot I$$

# Capacitor

Can store electrical charge (Q) in proportion to an applied voltage (U), the linear relationship between the two describes the capacitance (C).

$$C = \frac{Q}{U}$$

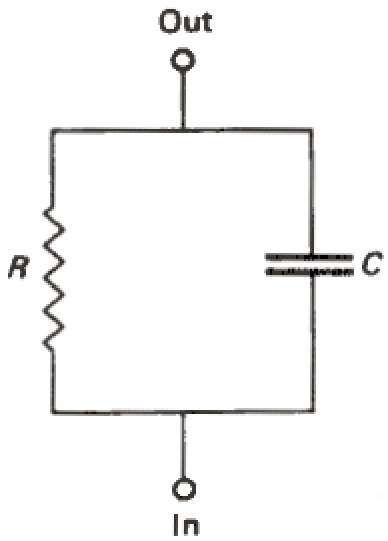
$$U = \frac{1}{C} \cdot Q$$

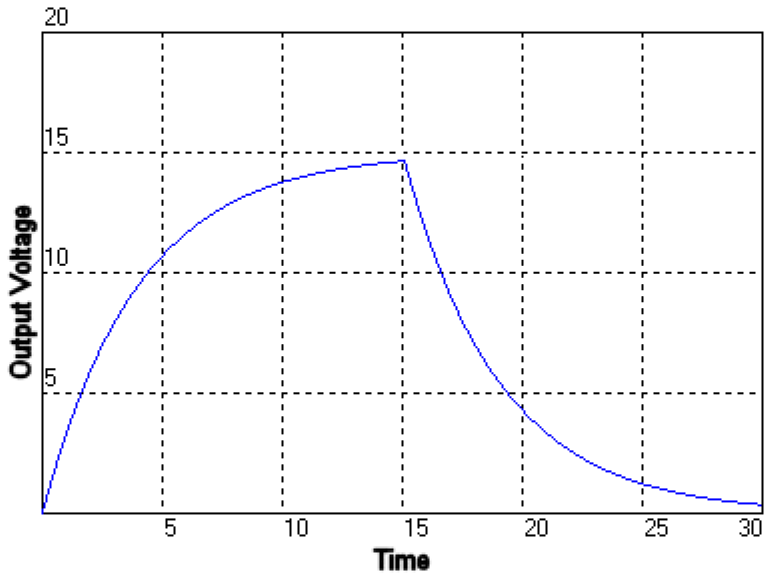



Electronics

Biology

# Bringing the two together




Since they can store charge, capacitors act as 'buffer', slowing voltage changes down. Any change in the voltage across a capacitor needs to move charges. The larger the resistance through which the charges have to flow the slower this is happening.

# Differential Equation

Connecting a capacitor and a resistor in the manner shown in the previous slide means that the capacitor discharges through the resistor. The voltage across the two is always equal.

The loss of charge is proportional to the remaining charge and to 1/RC.

This is a first order differential equation – compare to radioactive decay. The solution to this equation in the case ot the RC circuit is shown below – an exponential decay from an initial maximum $Q_0$ with a time constant RC.

$$U = RI$$

$$U = \frac{1}{C} Q$$

$$RI = \frac{1}{C} Q$$

$$I = -\frac{dQ}{dt}$$

$$-R \frac{dQ(t)}{dt} = \frac{1}{C} Q(t)$$

$$\frac{dQ(t)}{dt} = -\frac{1}{RC} Q(t)$$

$$Q(t) = Q_0 e^{-\frac{t}{RC}}$$

# What this Means for Neurons

- Changing the membrane voltage of neurons requires the charging or discharging of the membrane capacitance.
- The larger the capacitance and the higher the resistance the slower this process is happening.
- In order to speed things up the capacitance has to be reduced or the membrane resistance has to come down.

$$100\,pF$$

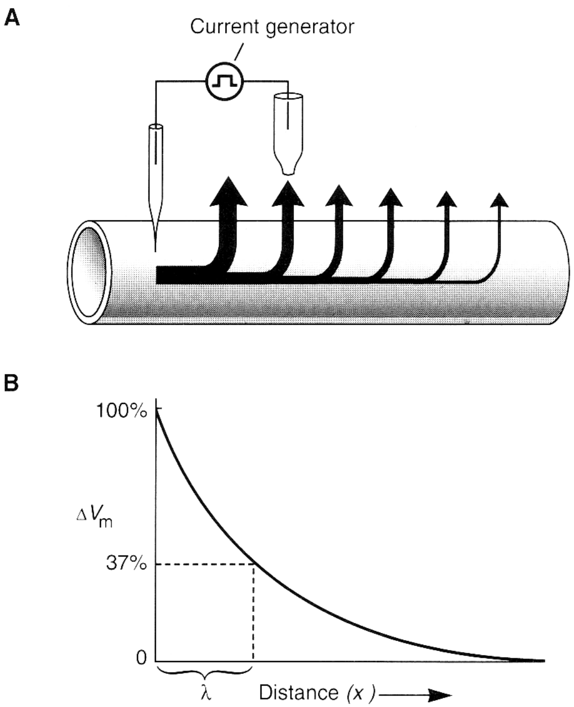
$$500 M\Omega$$

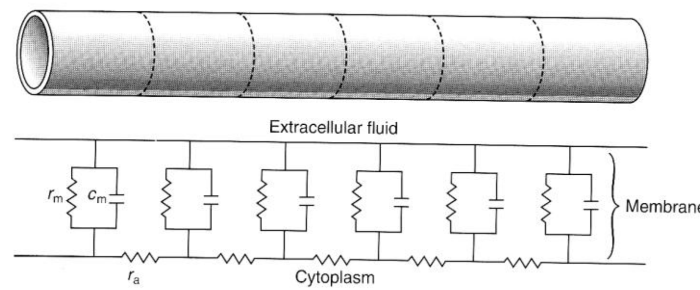
$$100 \cdot 10^{-12} C/V \cdot 500 \cdot 10^{6} Vs/C$$

$$50000 \cdot 10^{-6} s$$

$$50 ms$$

# Leaky Cables



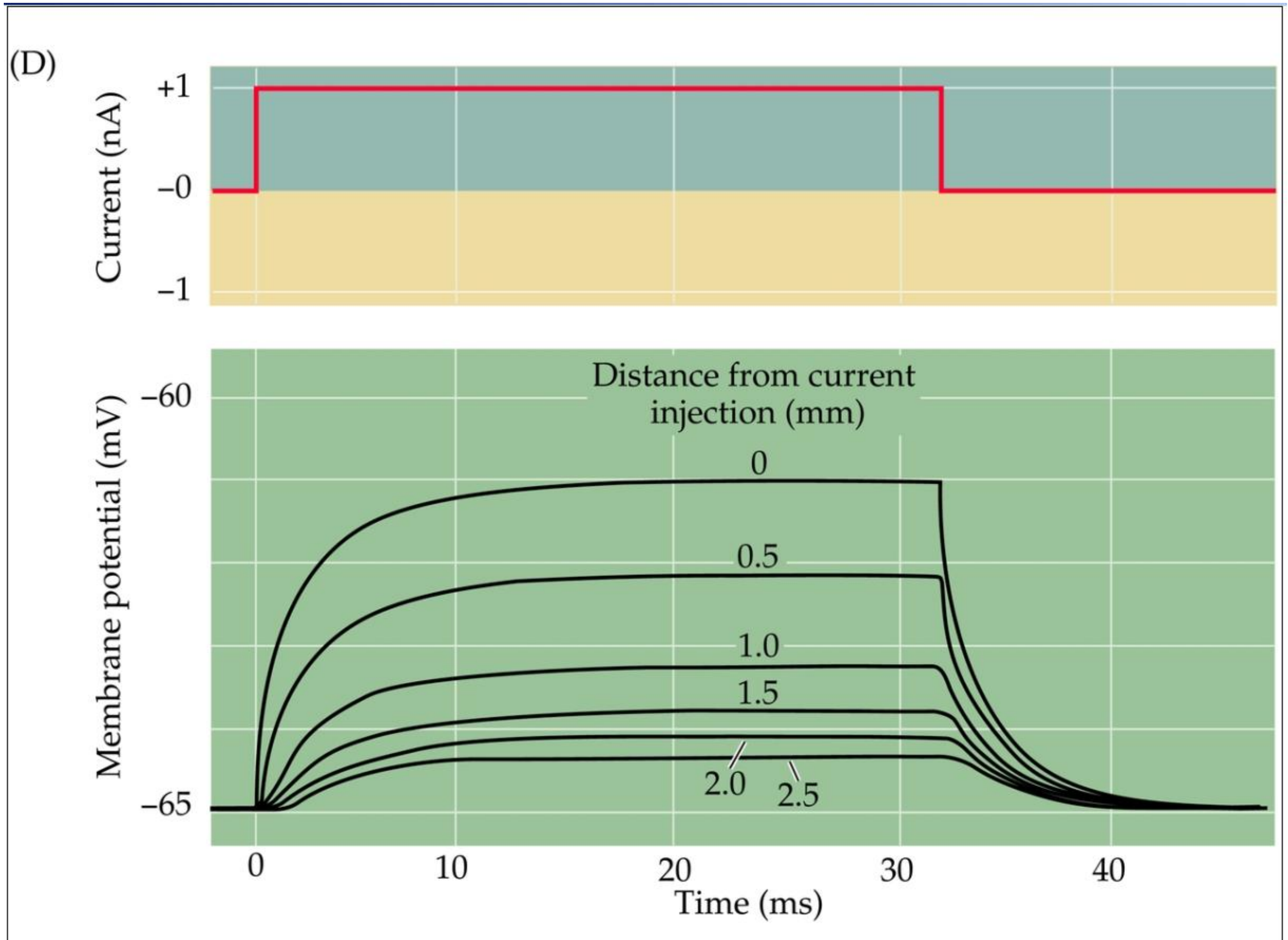




λ is typically between 0.1 bis 1 mm

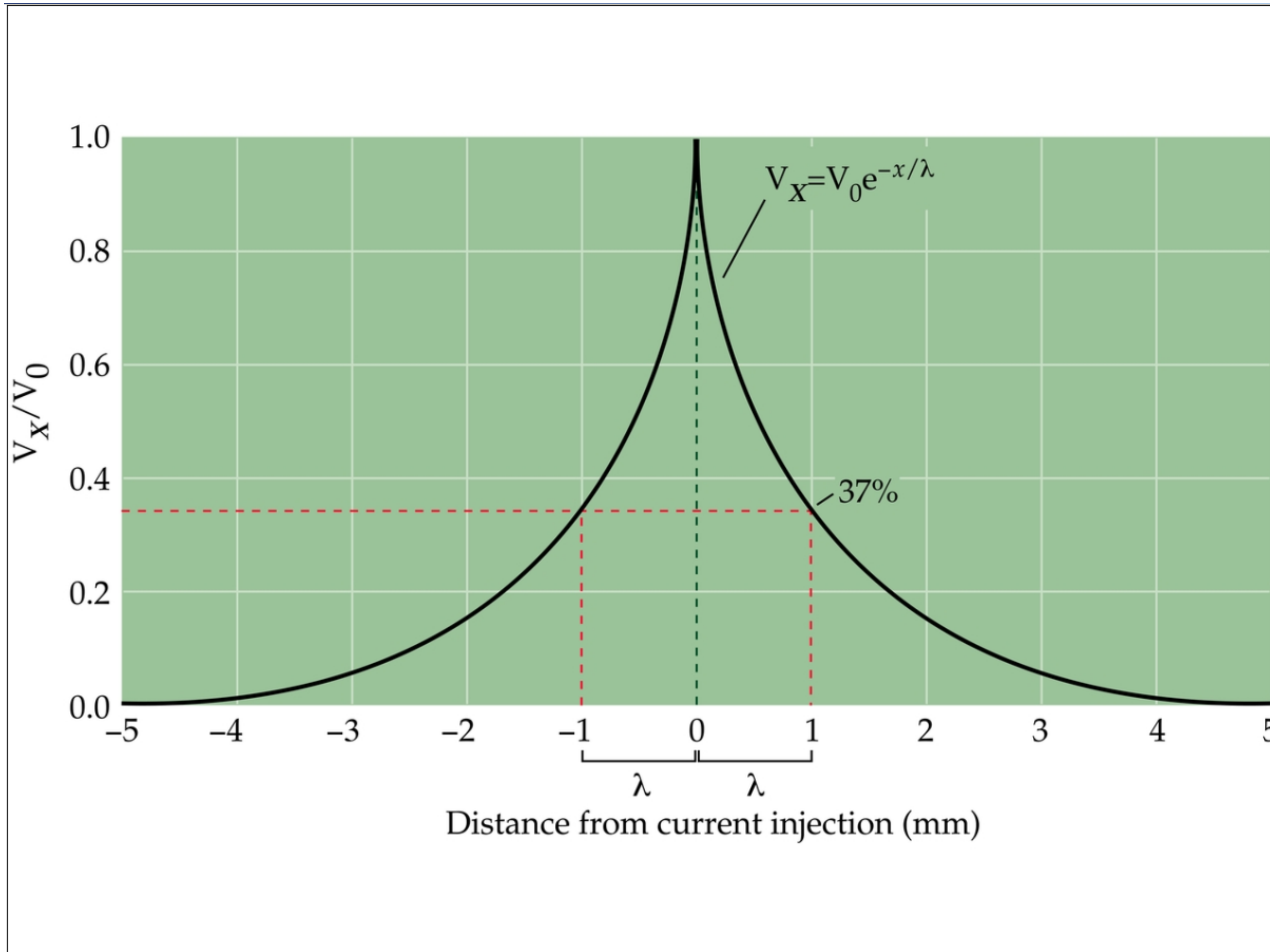
$$\lambda = \sqrt{\frac{r_{membrane}}{r_{longitudinal}}}$$

The length constant increases the smaller the longitudinal resistance gets. This is mostly a function of the diameter of the fiber. The thicker the fiber the larger the length constant.

# Loss of Signal



# Length Constant



# Adding Capacitive Effects

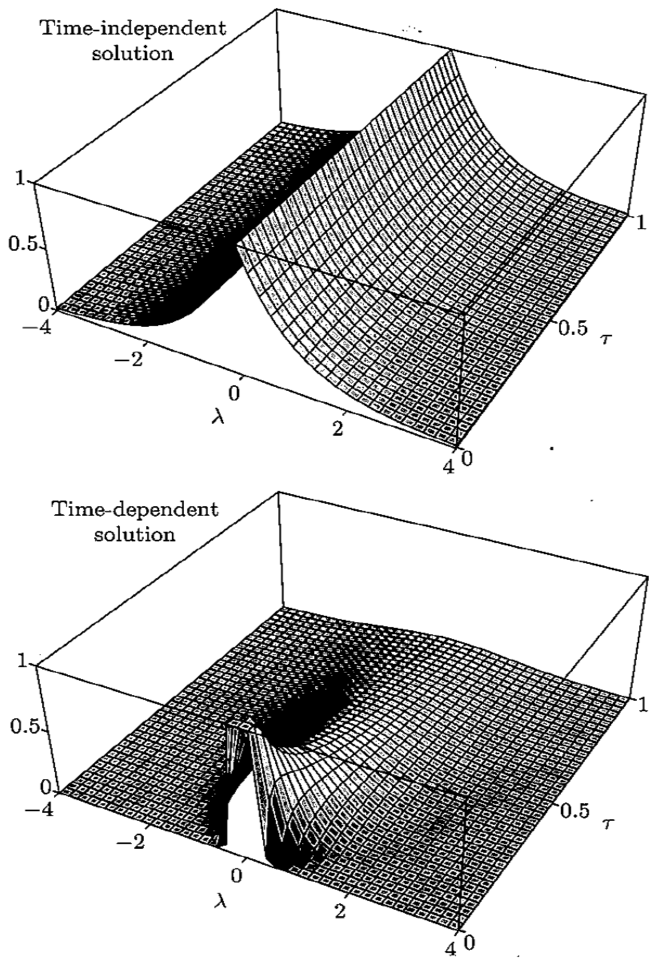



**Figure 3.25** Comparison of waveforms of the time-independent (upper panel) and time-dependent (lower panel) solutions of the cable equation in response to point current sources. The membrane potential responses are shown normalized. The upper panel shows the function $e^{-|\lambda|}$, and the lower panel shows the quantity $(1/\sqrt{4\pi\tau})e^{-\lambda^2/4\tau}e^{-\tau}$, both plotted versus $\lambda$ and $\tau$.

It is like sending a stream of water down a leaky and elastic garden hose.

It is like sending a pulse of water down a leaky and elastic garden hose.

page 124 - Cellular Biophysics V2 - T.F. Weiss, MIT Press 1996

# Summary

- Passive neuronal membranes behave like circuits composed of resistors and capacitors.
- The membrane potential can not change faster than the time constant.
- In the passive mode, neuronal processes conduct signals with a loss, depending on the membrane resistance and the longitudinal resistance.